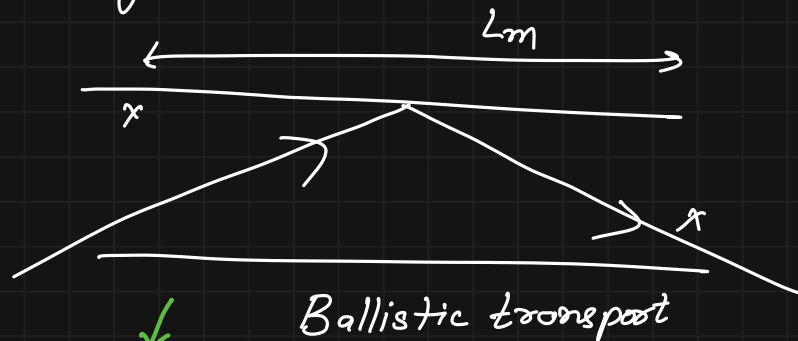


Careful of the terminology used in this field

Regimes
of transport

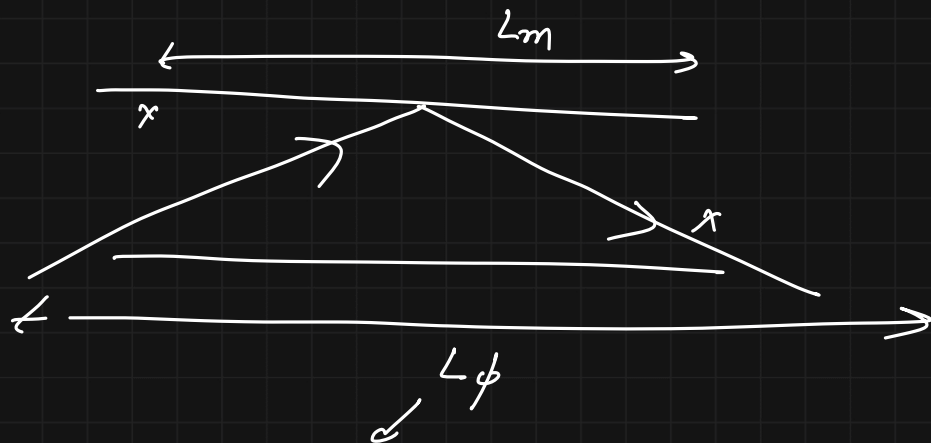


But very few scatters



we want to have a ballistic (clean) systems

→ transport regime where



* L_ϕ is large → scattering is only at boundaries

∴ To study diff regimes of ϕ . transport, we need to keep in mind
→ inelastic scattering lengthscale

$L_m, L_\phi, L_s, \lambda_{TF}$

elastic scattering length

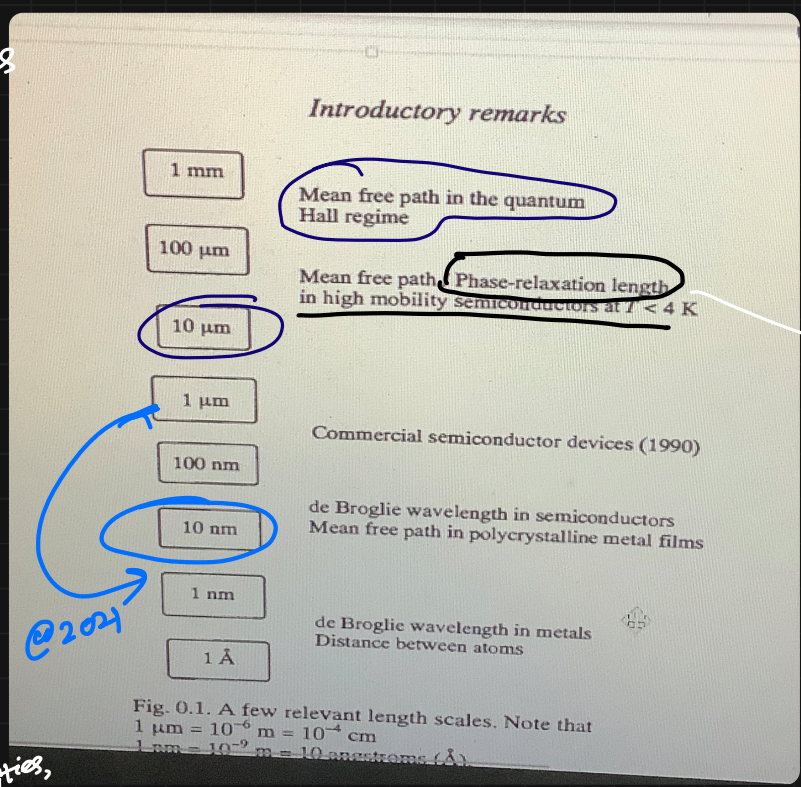
we'll assume a "phase coherent" & ballistic ("quasi-ballistic") transport.

mean free path depends on sample's frozen impurities

for metals, L_m & L_ϕ are rel. short.

ring expt $\rightarrow L_\phi \sim \text{few } \mu\text{m}$

coherence length $\rightarrow$ magnetic impurities, magnons,



"why call it relaxation?"

explanation for the gate (1st lecture) expt

~~experiment~~ experiment.

Transvers modes in outband

$$\left[E_s + \frac{(\hbar v + eA)^2}{2m} + U(y) \right] \psi(x,y) = E \psi(x,y)$$

$B = B \hat{z}$

$A = -\hat{x} B y \rightarrow a_x = -B y \leftarrow \text{choice of gauge}$

$$\left[E_s + \frac{(p_x + eBy)^2}{2m} + \frac{p_y^2}{2m} + U(y) \right] \psi(x,y) = E \psi(x,y)$$

1ex

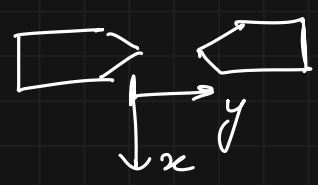
Fig. 0.1. A few relevant length scales. Note that $1 \mu\text{m} = 10^{-6} \text{m} = 10^{-4} \text{cm}$, $1 \text{nm} = 10^{-9} \text{m} = 10 \text{ angstroms } (\text{\AA})$.

we look at diff cases of confining potential

Gate mediated potential

basically gate effects this

Quadratic confinement



$$\psi(x,y) = \frac{1}{\sqrt{L}} e^{ikx} \chi(y)$$

$$\left[E_s + \frac{(\hbar k + eBy)^2}{2m} + \frac{p_y^2}{2m} + u(y) \right] \chi(y) = E \chi(y)$$

$$u(y) = \frac{1}{2} m \omega_0^2 y^2$$

now set $B=0$

$$\left[E_s + \frac{\hbar^2 k^2}{2m} + \frac{p_y^2}{2m} + u(y) \right] \chi(y) = E \chi(y)$$

$$u(y) = \frac{1}{2} m \omega_0^2 y^2$$

$$\left[E_s + \frac{\hbar^2 k^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2} m \omega_0^2 y^2 \right] \chi(y) = E \chi(y)$$

$$\chi_{n,p}(y) = u_n(q_y) \quad q_y = \sqrt{m \omega_0 / \hbar} y$$

$$E_{n,k} = E_s + \frac{\hbar^2 k^2}{2m} + (n + 1/2) \hbar \omega_0$$

offset
(rel. to band
edge)

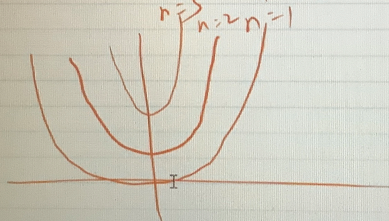
parabolic
disp

band #

$$E_{n,k} = E_s + \frac{\hbar^2 k^2}{2m} + (n + 1/2) \hbar \omega_0$$

$$u_n(y) = e^{-q^2/2} H_n(q)$$

$$v(n,k) = \frac{1}{\hbar} \frac{\partial E}{\partial k} = \frac{\hbar k}{m}$$

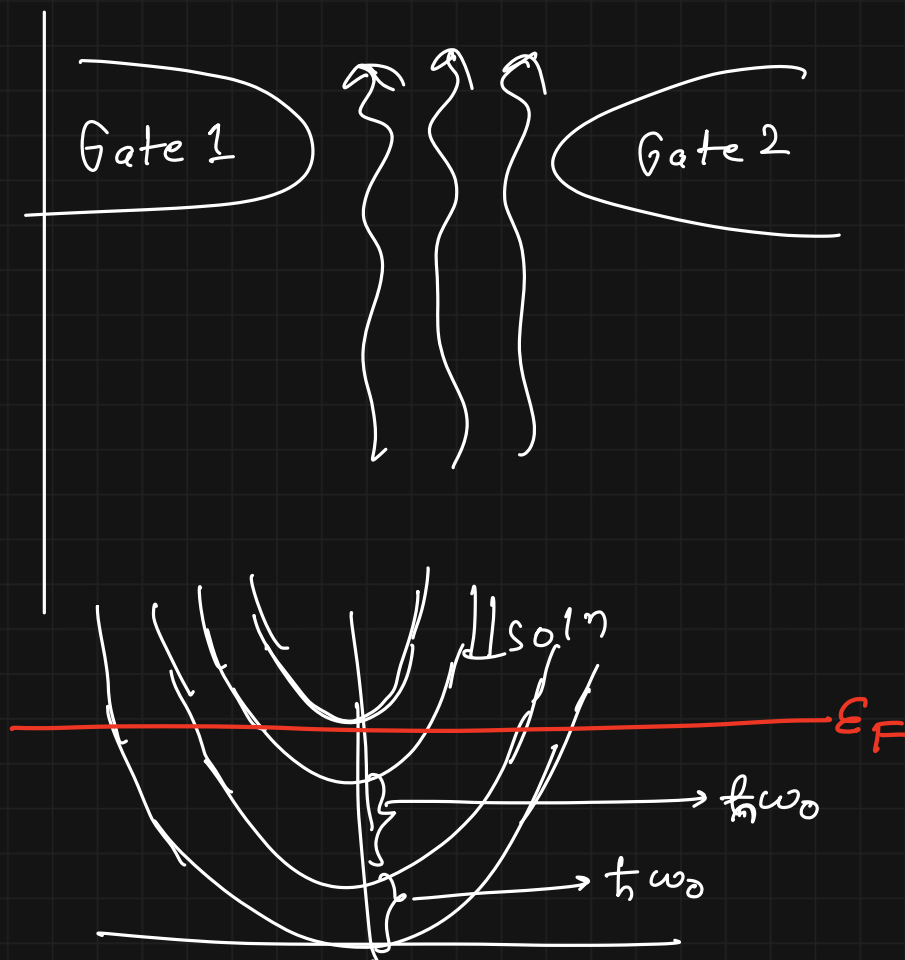


"Sub-bands"

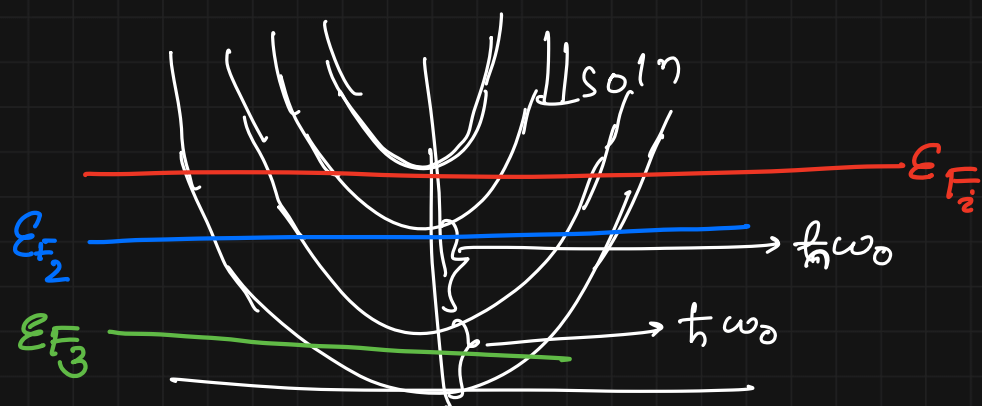
non-zero
group
velocity

"paper by von Wees"

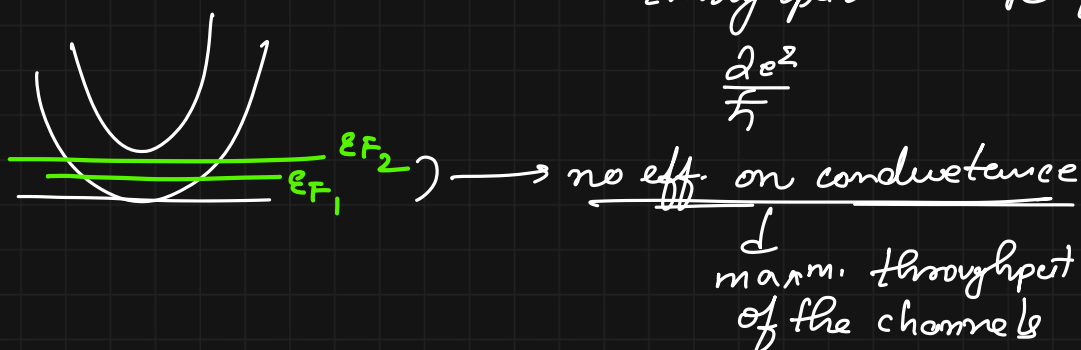
Back to
paper



as you change V_{gate} , the fermi energy gets tuned



as $E_F \downarrow$, # channels reduces $\rightarrow$ # of modes reduces & each mode has a max. throughput voltage of



$$\frac{2e^2}{h}$$



Q. Hall

→ no confinement → $\vec{B} \rightarrow 0$

$U=0$ & $B \neq 0$ (Dropping q_y as first approximation)

$$\left[E_s + \frac{p_y^2}{2m} + \frac{(eBy + \hbar k)^2}{2m} \right] \chi(y) = E \chi(y)$$

$$\left[E_s + \frac{p_y^2}{2m} + \frac{1}{2} m \omega_c^2 (y + y_k)^2 \right] \chi(y) = E \chi(y)$$

$y_k = \frac{\hbar k}{eB}$ $\omega_c = \frac{eB}{m}$ → spring const. is tuned

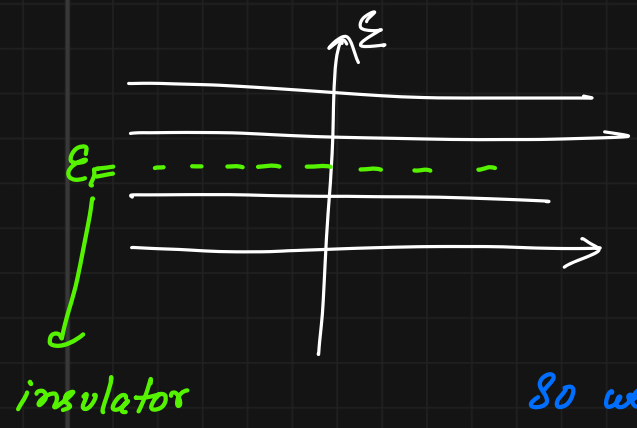
parabola centered at $y = -y_k$

$y_k = \frac{\hbar k}{eB}$ → shift

$$E(n, k) = E_s + (n + 1/2) \hbar \omega_c$$

$$\chi_{n,k}(y) = u_n(y + y_k)$$

$$q_y = \sqrt{\frac{m \omega_c}{\hbar}} y \quad q_k = \sqrt{\frac{m \omega_c}{\hbar}} y_k$$

$$\eta E(k) = E =$$


$$\chi_\eta = \frac{1}{\hbar} \frac{\partial E}{\partial k} = 0$$

"insulator"

So we started with a 2D "conductor",
we end up with an insulator

B +
confining
potential

confined $U \neq 0$ & $B \neq 0$

$$\left[E_s + \frac{p_y^2}{2m} + \frac{(eBy + \hbar k)^2}{2m} + \frac{1}{2} m \omega_c^2 y^2 \right] \chi(y) = E \chi(y)$$

$$q_y = \sqrt{\frac{m \omega_c}{\hbar}} y \quad q_k = \sqrt{\frac{m \omega_c}{\hbar}} y_k$$

$\frac{eB}{m} = \omega_c$ $y_k = \frac{\hbar k}{eB}$

$$\left(E_s + \frac{p_y^2}{2m} + \frac{(m \omega_c y + \hbar \omega_c y_k)^2}{2m} + \frac{1}{2} m \omega_c^2 y^2 \right) \chi(y) = E \chi(y)$$

$$\left[E_s + \frac{p_y^2}{2m} + \frac{m \omega_c^2}{2} (y + y_k)^2 + \frac{1}{2} m \omega_c^2 y^2 \right] \chi(y) =$$

$\downarrow$ Parabolic $\downarrow$ Parabolic

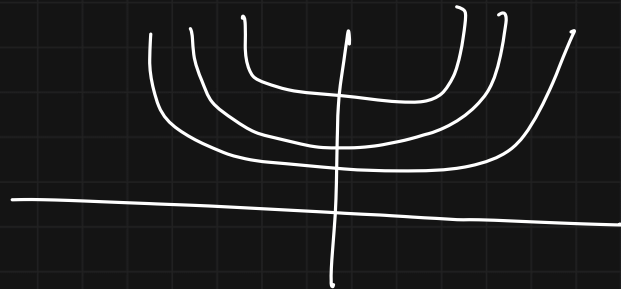
$$\left[E_s + \frac{p_z^2}{2m} + \frac{1}{2} m \frac{\omega_0^2 \omega_c^2}{\omega_c^2} y_k^2 + \frac{1}{2} m \omega_c \left(y_k \frac{\omega_c}{\omega_0} y_k \right) \right] \quad (k \neq 0)$$

$$\omega_c^2 = \omega_L^2 + \omega_s^2$$

$$\psi_{n,k} = \psi_n \left(y + \frac{\omega_s}{\omega_0} y_k \right)$$

$$y = \sqrt{\frac{m \omega_c}{\hbar}} y \quad y_k = \sqrt{\frac{m \omega_c}{\hbar}} y_k$$

$$E_{n,k} = E_n + \left(n + \frac{1}{2} \right) \hbar \omega_c + \frac{\hbar^2 k^2}{2m} \frac{\omega_0^2}{\omega_c^2}$$



$$y_k = \frac{\hbar k}{eB}$$

$$v(n,k) = \frac{\hbar k}{m} \times \frac{\omega_0^2}{\omega_c^2}$$

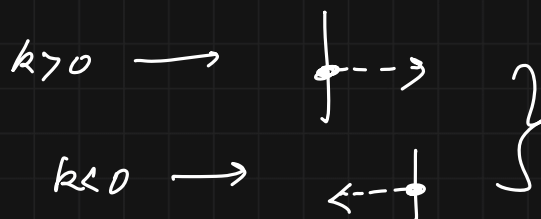
non-zero

$\downarrow$
diff from Q.Hall system

1st hint that
having a boundary
changes the
transport prop.

$$y_k = \frac{\hbar k}{eB}$$

$\hookrightarrow$ offset for wavefⁿ in y-direction



$v_k \propto y_k$

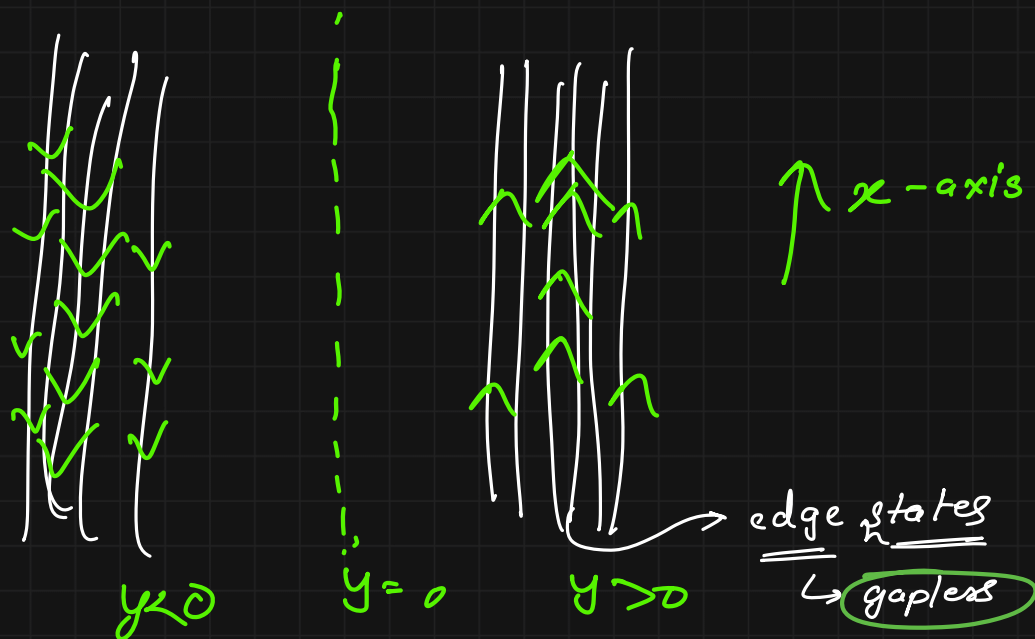
$\rightarrow \therefore$ for $k > 0 \rightarrow v_k > 0$

$y_k > 0 \rightarrow$ shifted
to right

$\rightarrow$ for $k < 0 \rightarrow v_k < 0$

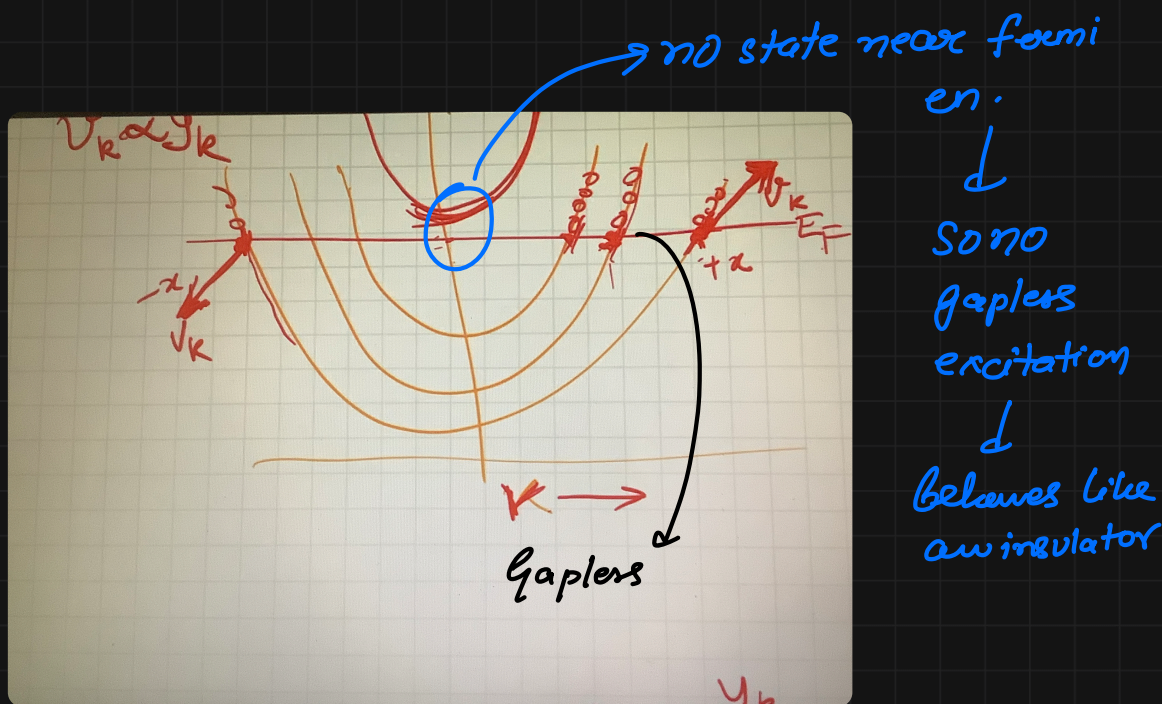
$y_k < 0 \rightarrow$ shifted to
left

Top view of ϕ Hall



→ move in opposite direction.

What happens in bulk?

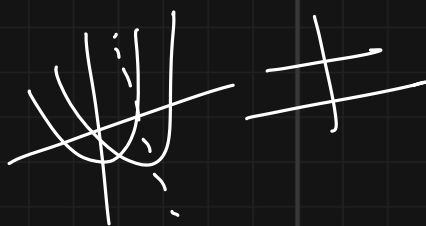


→ at the level of BTE, $\delta f = \frac{-q\tau E}{\hbar} \partial_k f$

How do I explain the Bulk "insulator"ness

within BTE?

Fermi energy



semi-classical
ideas

